1.

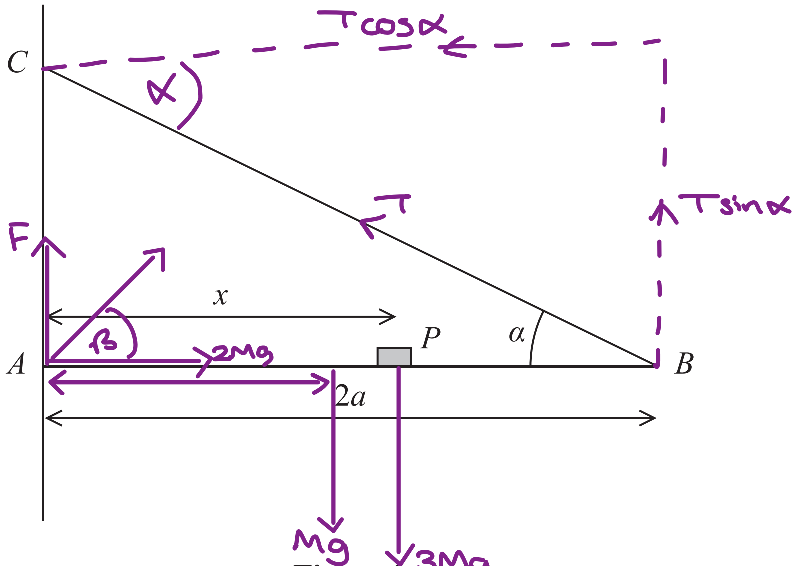


Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

DO NOT WRITE IN THIS AREA

a) Moments about A — 1

$(a \times Mg) + (x \times 3Mg) - (2a \times T\sin\alpha) = 0$

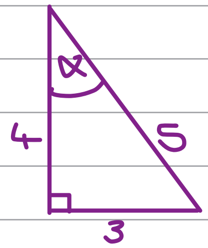
$aMg + 3xMg - \frac{6Ta}{5} = 0$

$\frac{6Ta}{5} = aMg + 3xMg$ — 1

$\frac{6Ta}{5} = Mg(a + 3x)$

$Ta = \frac{5Mg(3x+a)}{6}$

$T = \frac{5Mg(3x+a)}{6a}$ — 1



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

b) $R(\leftarrow)$:

$T\cos\alpha - 2Mg = 0$

$T\cos\alpha = 2Mg$

$\frac{5Mg(3x+a)}{6a} \times \frac{4}{5} = 2Mg$ — 1

$\frac{20Mg(3x+a)}{30a} = 2Mg$

$20Mg(3x+a) = 2Mg \times 30a$

$20(3x+a) = 60a$

$3x + a = 3a$

$3x = 2a$

$x = \frac{2}{3}a$ — 1

c) Moments about B ↺ - 1

$$(2a \times F) - (a \times Mg) - (2a - x)(3Mg) = 0$$

$$2aF = aMg + 6aMg - 3xMg$$

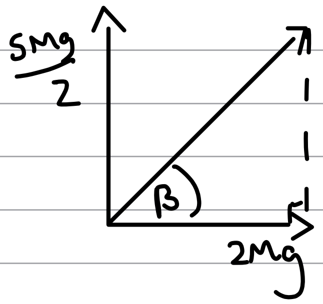
$$2aF = 7aMg - 3xMg \qquad \text{(from b))} \quad // x = \tfrac{2}{3}a$$

$$2aF = 7aMg - 2aMg$$

$$2F = 7Mg - 2Mg \quad - 1$$

$$F = \frac{7Mg - 2Mg}{2}$$

$$F = \frac{5Mg}{2} \quad - 1$$



$$\tan\beta = \frac{\frac{5Mg}{2}}{2Mg} \quad - 1$$

$$\tan\beta = \frac{5}{4} \quad - 1$$

d) $T \leq 5Mg$

$$\frac{5Mg(3x + a)}{6a} \leq 5Mg \quad - 1$$

$$5Mg(3x + a) \leq 5Mg \times 6a$$

$$3x + a \leq 6a$$

$$3x \leq 5a$$

$$x \leq \frac{5}{3}a \quad - 1$$

P must be no further away from A than $\frac{5a}{3}$. - 1

2.

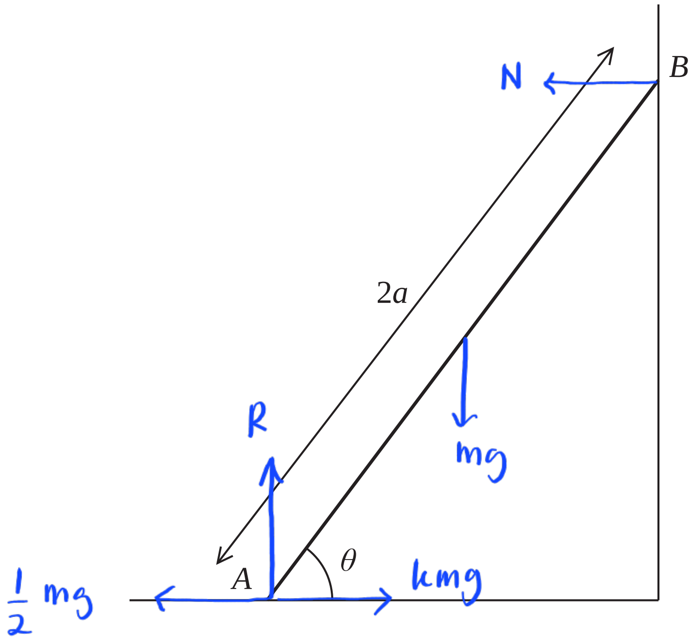


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k.

(5)

(a)

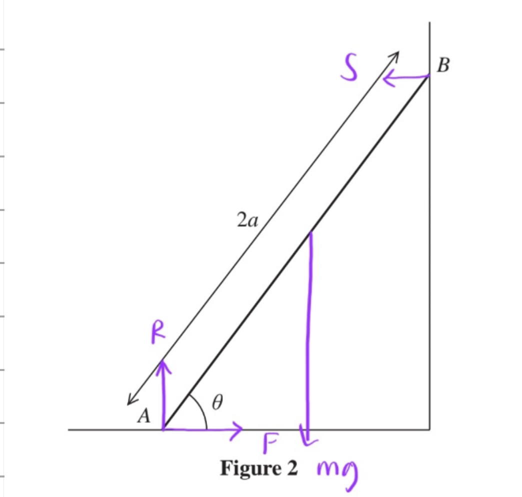


Figure 2

moment equation at A = $mg \times a\cos\theta = S \times 2a\sin\theta$ — (1)

Vertically : $R = mg$ — (2)

Horizontally : $F = S$ — (3)   substitute (2) and (3) into (1)

$\therefore \quad Ra\cos\theta = 2Fa\sin\theta$

$$F = \frac{Ra\cos\theta}{2a\sin\theta}$$

$$F = \frac{1}{2}R\cot\theta$$

However, $F \leqslant \mu R$ which means : $\frac{1}{2}R\cot\theta \leqslant \mu R$

$$\mu \geqslant \frac{1}{2}\cot\theta$$

the μR should be more than F because the beam is at rest.

(b)

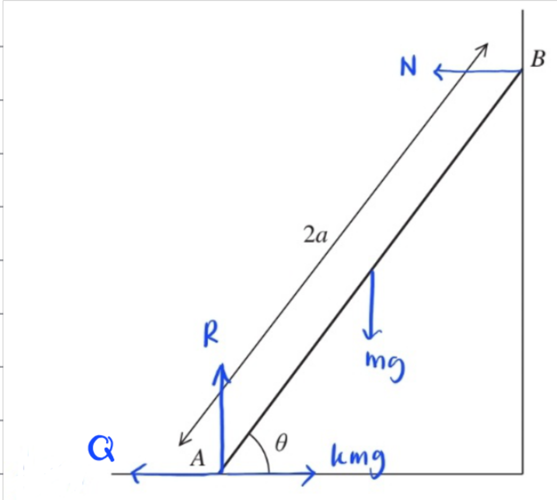


Figure 2

Moments equation at A : $mg \times a\cos\theta = N \times 2a\sin\theta$ ①

Vertical forces : $R = mg$ ②

Horizontal forces : $N + Q = kmg$

However, $Q = \mu R$ (limiting equilibrium) : $Q = \frac{1}{2}mg$

$\mu = 1/2$ (given in question)

$R = mg$ derived from ②

$\therefore N + Q = kmg$

$N + \frac{1}{2}mg = kmg$

$N = kmg - \frac{1}{2}mg$

$\therefore N = kR - \frac{1}{2}R$ — ③

Substitute ② and ③ into ①

$R\cancel{a}\cos\theta = (kR - \frac{1}{2}R) \times 2\cancel{a}\sin\theta$

$\cancel{R}\cos\theta = 2\cancel{R}(k - \frac{1}{2})\sin\theta$

$$\cos\theta = 2\sin\theta\left(k - \frac{1}{2}\right)$$

$$\frac{\cos\theta}{2\sin\theta} = k - \frac{1}{2}$$

$$\frac{1}{2}(\cot\theta) = k - \frac{1}{2}$$

Given, $\tan\theta = \frac{5}{4}$

$$\frac{1}{2}\left(\frac{1}{\tan\theta}\right) = k - \frac{1}{2}$$

$$\frac{1}{2}\left(\frac{4}{5}\right) = k - \frac{1}{2}$$

$$\frac{4}{10} = k - \frac{1}{2}$$

$$k = \frac{4}{10} + \frac{1}{2}$$

$$k = \frac{9}{10}$$

$$k = 0.9 \quad \#$$

1

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA